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To cite this article: V Velázquez *et al* 2013 *J. Phys.: Conf. Ser.* **475** 012014

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Self similitude in the power spectra of nuclear energy levels

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Abstract. Protons and neutrons interact inside the nuclei with a non integrable interaction. It is shown that is possible to reproduce the energy statistics of nuclear energy levels analyzing the fluctuations of an ensemble of nuclear states produced by random interactions. In this contribution we analyze the characteristics of the fluctuations and distributions of the energy levels in order to understand the independent scale nature of the $1/f$ -noise nuclear energy levels.

1. Introduction

The statistics of energy nuclear levels has been an important topic in the study of quantum chaos. In quantum mechanical systems with classical analogue chaotic ones [1], it has not yet been observed the “butterfly effect” in any quantum observable. Then, the research in quantum chaos in the latest years has been focused in search of convincing traces of quantum chaos. Although there is not satisfactory definition of the meaning of quantum chaos in comparison with the classical chaos [2], theoreticians have found some *signatures* of this behavior in quantum systems. In particular in the atomic nuclei, the energy levels distributions have been classified in one of the main statistics limits: Poisson or Wigner distributions [3]. These limits are well related with integrable and non integrable interactions, respectively.

What is very amazing is that an ensemble of protons and neutrons, interacting under the influence of a very complicated force built from the nuclear force of all nucleons, does not display a “nuclear butterfly effect” in any observable: energies, BE2 transition strenghts, masses, quadrupole moments, etc. We know that the nuclear interaction is a non integrable force, then the nuclear dynamics must evolve like a complex system. Maybe, we must accept that the complexity in quantum system is manifested as fluctuations in the interference of states of a given Hilbert space. Thus, some interesting signatures of quantum chaos are centered in the fluctuations analysis [4, 5]. In this contribution we start from the premise that a quantum system like the nucleus is a complex system and the interest is placed in the applications of their statistical properties to classical complex systems, leaving the usual approach of seeking traces



of quantum chaos. The goal of the present work is to show that if a classical complex system has a quantum analogue, then we can define analogue interactions which are integrable and non-integrable, as implied by the matricial calculus. A very interesting example of this behavior introduced by Berry [6], is that the Riemann zeros (then, the prime numbers) could be obtained from a eigenvalues problem.

2. Nuclear shell model energies

We use the nuclear shell model in order to describe theoretically the nuclear dynamics. In this model an interaction (in its simplest approximation) is composed by one average interaction part and another coming from the many body interaction, both named here as monopole and multipole interactions, respectively

$$\hat{H}_{SM} = \hat{H}_{monopole} + \hat{H}_{multipole} . \quad (1)$$

In its simplest approximation, the monopole part is based in the harmonic oscillator potential, doing a mimic of a mean field in which each nucleon interacts independently; the multipolar part can be approximated by a few two body interaction terms, neglecting higher-body orders. Then, selecting a good interaction \hat{H}_{SM} , a large enough Hilbert space, and a dependable computational method (the Lanczos algorithm), we can study the behavior of some observable in function of an set of initial conditions.

We propose the ^{48}Ca nucleus because its energy distribution has been studied by several methods and it has been demonstrated its complexity. In the present contribution, we are interested in the study of the evolution of complexity and its self similitude. The interaction is taken as a harmonic oscillator mean field added with the monopole part of the KB3 [7] realistic interaction. This interaction has been derived by minimally modifying the monopole strength in the original Kuo-Brown interaction [8]. The two body interaction (multipole part) is composed by an ensemble of random interactions (two body random ensemble or TBRE) [9]. The Hilbert space selected is the full fp shell: it is the harmonic oscillator shell for $\eta = 3$, which has demonstrated to be a very good truncation to study this nucleus. The shell model calculation is performed with the code ANTOINE [10], which works in the m-scheme. As we are interested in the evolution of the energy levels statistic, and therefore in its correlation, we use the method of power spectra [11, 12, 13, 14]. We decided to analyze the correlation of the 1627 $J^\pi = 3^+$ states in the fp shell for the ^{48}Ca . We can select an average amplitude interaction if we select a TBRE, whose matrix elements follow a normal distribution with a width near to the realistic matrix elements. In the case of the KB3 interaction we have $\sigma = 0.6$ [15]. In the present calculations, we have selected the widths $\sigma = 0.2$ and $\sigma = 0.01$ in order to study the evolution in the correlation and the scale invariance. The first value ($\sigma = 0.2$) gives the width of the energy levels distribution of the KB3 interaction (this corresponds with the realistic value), while $\sigma = 0.01$ is twenty times smaller [16].

Figure 1 shows the ordered distribution of the 1627 eigenvalues. This is a global pattern that follows the energies for a wide interval of interactions. In fact, this sequence of ordered energies appears in a wide range of phenomena of different topics of research [18]. This class of distributions can be outlined by the binomial [19] or beta [18] (among others) descriptions. The “s” stylized geometry is a signature of a diagonalized matrix. The extreme values are pushed towards up and down the mean value. The central eigenvalues come from the several mixed interactions, thus the predictability will obviously depend on the nature of the interaction. If the interaction is non-integrable, the fluctuations around this “s” will be chaotic, and for an integrable interaction (clasically integrable), they will be non-chaotic [12].

We are interested in the fluctuations around this mean behavior. With the unfolded fluctuations [12], we are ready to carry out the power spectra analysis

$$S(f) \approx 1/f^\beta . \quad (2)$$

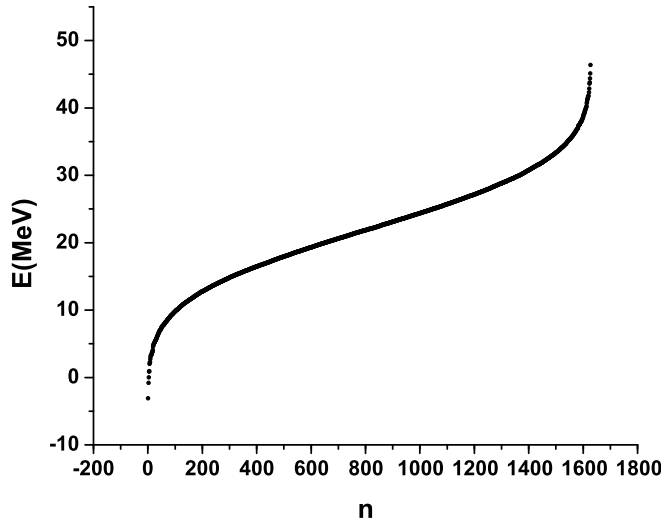


Figure 1. Energy of the 1627 nuclear levels with $J^\pi = 3^+$ in ^{48}Ca . The x -axis is labeled with the ordered number level n . The interactions used is a TBRE with strengths obtained from a gaussian distribution with width $\sigma = 0.2$.

If the power spectra follow a power-law, then it suggests that the system could be scale invariant, particularly the $1/f$ power spectra can be also related with quantum chaos [11].

3. Unfolded energies

The unfolding procedure consists in the subtraction of the mean behavior of the energy distribution. Then, we rename the energies to emphasize the mapping.

$$E_i \rightarrow \epsilon_i \equiv \bar{N}(E_i), \quad (i = 1, \dots, N) \quad (3)$$

where $\bar{N}(E_i)$ is a smooth function fit of the staircase-like cumulative density function $N(E_i)$ [2, 17]. The nearest neighbor spacing is calculated as $s_i = \epsilon_{i+1} - \epsilon_i$, $i = 1, 2, \dots, N - 1$. The spectrum fluctuations can be defined by

$$\delta_n = \sum_{i=1}^n (s_i - \langle s \rangle) = [\epsilon_{n+1} - \epsilon_i] - n \langle s \rangle. \quad (4)$$

The stochastic discrete function δ_n contains the deviations of the distance between the first and the $(n + 1)$ -th unfolded states, related to the corresponding distance in a uniform (equally spaced) sequence having a unit level distance $\langle s \rangle = 1$. The sequence (4) can be interpreted as a discrete “times series”. In order to understand the self-similitude behavior, we compare the power spectrum of the several times series obtained for sets of energies $J^\pi = 3^+$ in ^{48}Ca obtained with TBRE interactions at two very different intensities.

From the power spectrum analysis, we find that the fluctuation around the distributions in the Figure 1 is chaotic, that it is $\beta = 1.02$; a result that has been widely reproduced in the literature [20].

4. Self similitude in the chaotic behavior

We carry out the diagonalization procedure for the same system ^{48}Ca with a weaker interaction, twenty times smaller ($\sigma = 0.01$) than the former. The Figure 2, it is shown the ordered energy distribution of the 1627 levels with $J^\pi = 3^+$ angular momentum. A difference with the first case presented ($\sigma = 0.2$) is that in this case the sequence distribution is segmented. The matricial trend is kept, but structured. When the quadrupole-quadrupole interaction is small, the matrix

is composed by blocks, where each block corresponds with a subset of levels appearing bunched after the diagonalization. When $Q \cdot Q$ is large, the matrix is not built by blocks (not structured), because the non-diagonal matrix elements are very large. Now, we repeat the procedure of unfolding the power spectrum, that results in $\beta = 1.89$.

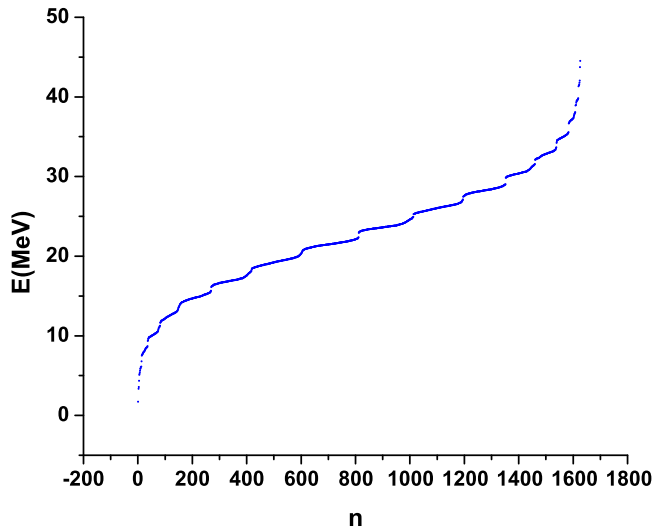


Figure 2. Energy of the 1627 nuclear levels with $J^\pi = 3^+$ in ^{48}Ca . The x -axis is labeled with the ordered number level n . The interactions used is a TBRE with numbers obtained from a gaussian distribution with width $\sigma = 0.01$. Note that the energy levels distributions are bunched.

A weaker interaction changes the statistics between neighboring energies, from chaotic towards thermal. Using the chromatics classification of noise fluctuations, these goes from pink to brown noise ($1/f^2$) when we reduce the intensity of the interaction. Our interaction is a shell model random one, with almost all two body terms that include integrable and non integrable terms as a quadrupole-quadrupole one. Then, for a β value greater than 1, the correlation between energies is greater than the chaotic correlation. This behavior appears intuitive. The mean phenomena is the interaction between states. Each final state is the result of several scattered positions in energy, where scattered positions signify avoided crossings between energy levels. In principle each state must feel the influence of all other states with the same symmetry, via the quadrupole-quadrupole interaction. In order to interpret the curve in Figure 2, is better to start from a weak interaction: in the Hamiltonian (1) the dominant term is the monopole part, because the two body interaction is neglected. Then we have the symmetry imposed by the mean field, and some degeneration is kept. This degeneration can be reinterpreted as bunched energy levels, and we know that bunched statistics (which means brown noise related with termal noise) can be understood like thermal statistics. Because $\hat{Q} \cdot \hat{Q}$ destroys this degeneration, the increase of its intensity will change the statistics, because the different bunches will mix themselves up to smooth the distribution curve as shown in Figure 1. In Figure 3, we select one bunch in the central part of the Figure 2, where it is evident the self-similitude in the power spectra. The power spectrum gives us $\beta = 1.18$. Its behavior is near to quantum chaos instead of thermal. Although the number of energies composing this plot is few, the trend predicts a $1/f$ -noise.

5. remarks and conclusions

Summarizing, for every intensity of the interaction we always get quantum chaos or $1/f$ noise at different scales of energy. In other words, for every strength of the quadrupole interaction, their effects will be those implied by nuclear configuration mixing, and this mixing will occur with few or lot of bunched levels. We always find $1/f$ noise, does not matter of the consideration of

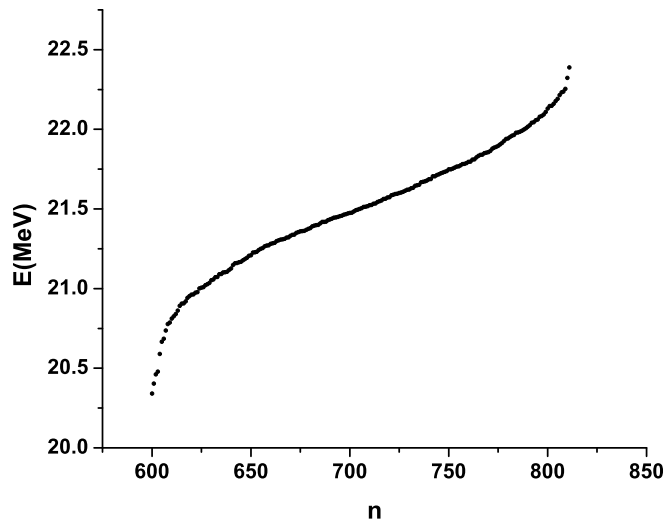


Figure 3. Energy levels distribution, detail from Figure 2.

complete set of levels or a subset of them. It is always present when the interaction contains a non-integrable component. The discussion can give some extra clues about the complexity and the unfolding technique. A smart unfolding on the bunched distribution in Figure 2 would give directly $1/f$ noise, but this does not mean that the thermal distribution does not have importance; nevertheless, noises are of physical interest. Then, from the analysis of the three Figures, we may get the following appreciations:

- (i) The energy distribution with inverted “s” geometry, indicates the presence of some interaction between states of the finite Hilbert space considered, in this case the fp -shell.
- (ii) The fluctuations about the inverted “s” distribution, indicate the nature of the interaction: $s(f) = \text{constant}$ and $\beta = 0$, then the interaction is integrable. In the non-integrable case, the fluctuations may have some correlation or anti-correlation.
- (iii) The strength of the interaction can change from one correlation to another, however the nature of the interaction will always stay in the final unfolding. Then, if the interaction contains some non-integrable part, the $1/f$ noise will appear in some scale.

Returning to the bunched energy distribution ($1/f^2$ noise), we could understand this pattern in a simple way: the interaction is so weak that its influence can not reach beyond certain set levels clustered by the mean field interaction, but inside of each cluster all states are mixed by the non-integrable interaction producing the local $1/f$ noise. As an example of this behavior, we can get the prime numbers distribution. If we plot the time series of the first 100000 prime numbers versus the natural numbers up to the greatest prime (see Figure 4), we can see that the prime numbers sequence follows a curve always below of the straight line of the natural numbers. In the context of the present work, the curve comes from a possible eigenvalue problem (from a matrix structure). If the prime numbers sequence would be infinite, the curve will never cross the straight line. Fluctuations around the bend were measured by M. Wolf [21]. Although in this Reference the author claims a $1/f$ noise, he found $\beta = 1.64$, then in the context of the present work, the fluctuations have some degree of correlation, towards bunching, and it is natural because the prime numbers structure includes the twin primes. Such twin primes add correlation to the fluctuations. The importance of these results might explain some another questions. The universality of the inverted “s” distribution [18] arising in systems of very

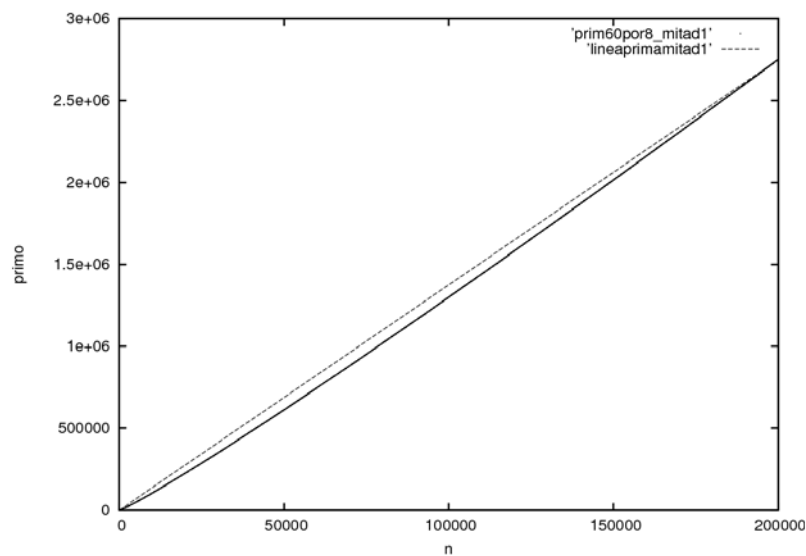


Figure 4. Time series of prime numbers plotted versus the natural numbers up to the maximal natural prime number. The primes curve can be related with the eigenvalues from a matrix problem.

varied nature, would be understood in terms of space configurations and interactions. In a very wide sense, we can say that if a community has interaction between their components, then an observable will be distributed over an inverted “s” geometry, modulated by the space in which it is contained, and it will have a noise controlled by the interaction nature.

Acknowledgments

We would like to thank to Facultad de Ciencias-UNAM for the sabbatical year grant of CEV. Work supported by PAPIIT-UNAM, and CONACyT-México.

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