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Quantum interference vs. quantum chaos in the nuclear shell model

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Abstract. In this paper we study the complexity of the nuclear states in terms of a two body quadrupole-quadrupole interaction. Energy distributions and eigenvectors composition exhibit a visible interference pattern which is dependent on the intensity of the interaction. In analogy with optics, the visibility of the interference is related to the purity of the states, therefore, we show that the fluctuations associated with quantum chaos have as their origin the remaining quantum coherence with a visibility magnitude close to 5%.

1. Introduction

Starting with the premise that every classical mechanics state (C) may be described by a subjacent theory based on quantum mechanics (Q), the transition from quantum to classical behavior and vice-versa deserves to be studied; see [1] for example. What are the characteristics of such a transition? What is lost or gained in this process? Is the transition a sudden shift between two phases or does it take place in a continuous manner? There are questions that are often raised regarding the bridge joining classical to quantum worlds. First of all, we have some clear ideas about how those systems behave in corresponding phases. We know that if a quantum system has access to a set of conditions its state is a linear superposition of all of those, for instance. This is known as the spectral decomposition principle [2]. Therefore, it is said that a physical state coexists in all the states of the base in which it has been described with a certain amplitudes of probability distribution. If this amplitudes of probability distribution is fixed, the state is considered a quantum one; if it is random (or mixed statistics) it is considered classical. This quantum coexistence of states permits that some phenomena such as auto-interference and quantum entanglement are properties without any classical analogies. In the $Q \rightarrow C$ transition,



the quantum state is converted into a classical state when coexistence cannot be traced back (this does not mean that it does not exist). According to Zurek [1] and other authors argument [3, 4, 5] this transition finds its origin in the interaction of the quantum ensemble with the surrounding environment. This phenomenon is known as decoherence. Decoherence is thus caused by the information exchange between the system (S) and its surrounding environment (E). When the environment is classical, the exchange will not be controlled and the quantum (fixed) amplitudes of probability distribution will be thermal, turning to random one, thus classical. In this paper, we show that this information exchange is taking place via an interaction. Every interaction process implies mixed quantum components. In the nuclear shell model framework, we suppose that each nucleon interact with a quantum environment, then, information is developed through two mechanisms: the number of times the nucleon interacts with the environment in an average time, and the interaction strength. We will use concepts such as quantum interference as a property of the states purity and show how this property is lost with the mixing of configurations. As a conclusion of this analysis we propose that quantum fluctuations of the nuclear energy levels, widely studied in the framework of quantum chaos (QC), find their origin in the residuum of quantum coherence, nearly totally lost due to multiple interactions between the states of the configuration space. Because we consider a quantum isolated environment our transition will be $Q \rightarrow QC$ For this goal we develop shell model calculations using a tunable two body quadrupole-quadrupole interaction.

2. The Model

The most important ingredient of any model is its interaction. In the nuclear shell model, the interaction is modeled as the sum of an mean interaction and an interaction of N bodies, respectively called monopolar (m) and multipolar (M).

$$\hat{H}_{SM} = \hat{H}_m + \hat{H}_M \quad (1)$$

The simplest representation of this sum is the field of an independent particle (monopolar part) plus the interaction of two bodies (multipolar part). In practice, the diagonal part of the interaction of two bodies is usually included in the monopolar part. We have selected a schematic interaction such that we can observe the influence of the mixed states on the $Q \rightarrow QC$ transition via the quadrupolar interaction parameter. We thus considered an interaction of two bodies composed of quadrupolar and pairing interactions. The monopolar interaction will be composed of the mean field of a harmonic oscillator including the diagonal part of the modified realistic Kuo and Brown interaction, KB3 [6].

$$\hat{H}_{SM} = \hat{H}_m - \chi \hat{Q} \cdot \hat{Q} + g \hat{P} \cdot \hat{P} \quad (2)$$

The pairing interaction coupling constant value will be set to $g = 0.46$ [7] and the quadrupolar interaction parameter will be swept within an acceptable interval. This interaction will then be applied to the ^{48}Ca nucleus problem in the fp shell. We will use the shell model code ANTOINE [8] with a Lanczos diagonalization process. In particular, we will calculate the statistics of 1627 states with $J^p = 3^+$. Figure 1 presents the energy sequences for each diagonalization of the Hamiltonian (2) with a dependency on the quadrupolar interaction intensity parameter χ . It can be seen that for lower values of χ the sequence is discontinued; it presents energy jumps. As the interaction of two bodies increases, the empty spaces fill themselves and the energy series curves are flattened. Figure 2 shows the distribution of energy levels for four quadrupolar force intensities: (a) $\chi = 0.01$; (b) $\chi = 0.04$; (c) $\chi = 0.07$ y (d) $\chi = 0.25$. The distribution of energies

present peaks that disappear as the interaction increases, as can be seen on these graphs. The peaks represent the energy leaps mentioned in the first series of figure 1 and are energy ensembles that have not been mixed with the energies of other peaks. The quadrupolar interaction is a long-distance interaction, but its range is limited at very low intensities. Therefore, each peak is a local distribution of energy in which interactions appear between energies of the sub-ensemble [11].

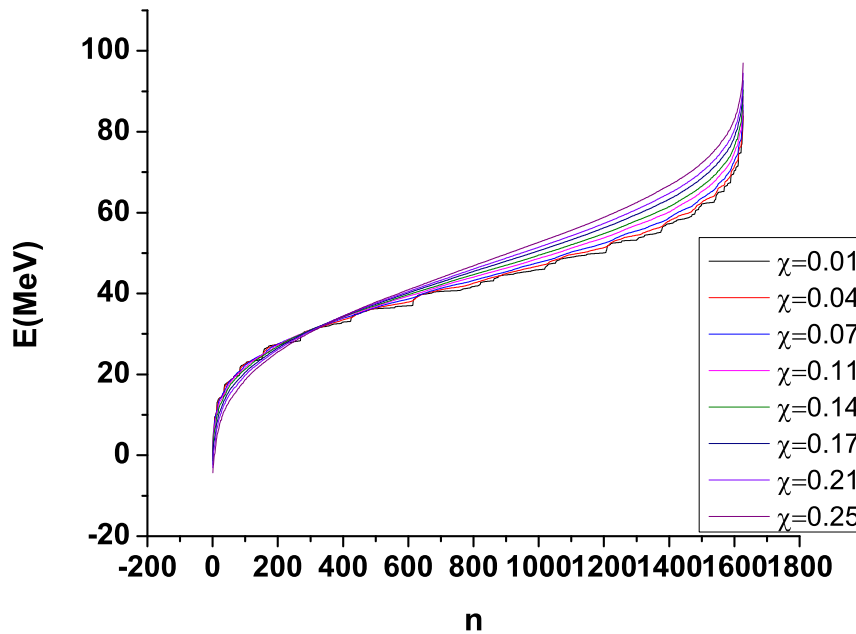


Figure 1. Energies of the 1627 states with $J = 3+$ in ^{48}Ca with dependence of the quadrupole-quadrupole parameter χ . For weak interactions is possible to observe jumps in the sequences of energies.

3. Energy interference

The 1627 states are constrained to certain spectral zones, which allows us to say that the spectrum present some form of organization. Energies are thus organized in sub-ensembles that share the common property that at low interaction intensities their particular states are not mixed with the states of another sub-ensemble. This can be interpreted as interference. In fact, the distribution of energies forms an interference pattern. The definition of interference indeed implies that it exists if the states that produce it are identical or at least present mutual coherence. The difference between the four graphs of figure 2a lies in their respective degree of coherence: it is decreases from figure 2a to figure 2d.

Technically, the nucleus state in the sub-space $J^p = 3^+$ can be written in a certain base $|k\rangle$:

$$|\psi^i\rangle = \sum_{k=1}^{1627} C_k^i |k\rangle \quad (3)$$

where each one of the energies E_i (i from 1 to 1627) E_i is a solution of:

$$\hat{H}|\psi^i\rangle = \bar{E}^i|\psi^i\rangle \quad (4)$$

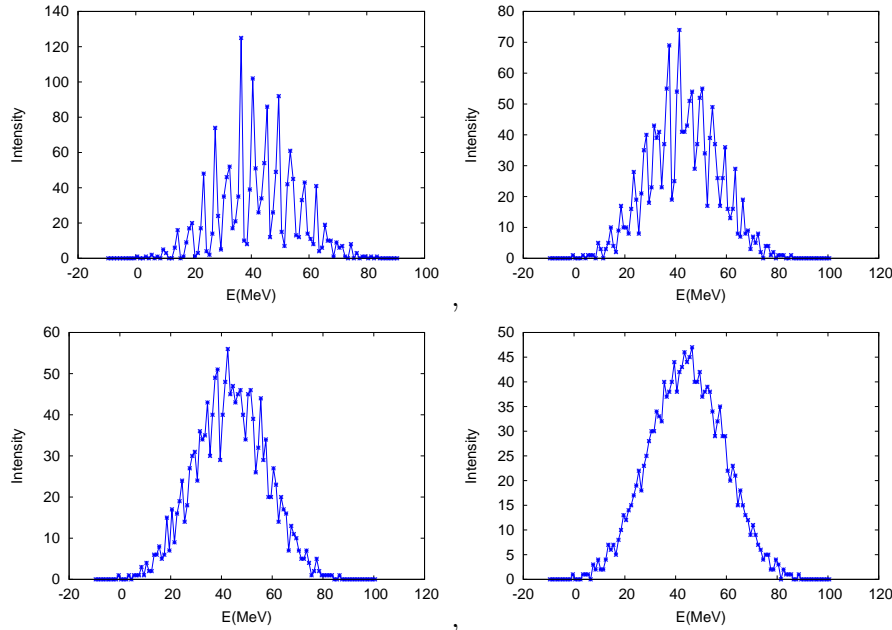


Figure 2. Energy distribution plots for four of the eight energy series showed in the Fig 1. : a) $\chi = 0.01$, b) $\chi = 0.04$, c) $\chi = 0.07$, y d) $\chi = 0.25$

if the states are not mixed statistically with each other. In this case we say the states are pure: $\bar{E}^i = E^i$ If the states are not pure, the energy of each state will be the mean of of the states energies E^i that compose the mixed statistics. The deviation around the mean is determined by the coefficients C_k^j . The maximum degree of purity that each state can present is $|C_k^i|^2 = \delta_{i,k}$. In principle, the mean field already broke the degeneration of the harmonic oscillator field; however, it is possible to say that the states maintain a certain degree or purity as gaps are observed in the energy density.

4. Power spectrum and visibility

As the quadrupolar interaction intensity increases the states mixing increases and the gaps gradually disappear. When the interaction increases up to one value similar to the force of the realistic interaction, the interference is reduced down to fluctuations (mainly visible around the maximum of the distribution, figure 2d.). The meaning of these fluctuations as the remainder of a certain coherence can be interpreted back (from distribution to spectrum) as the fluctuation of each energy around its mean. How can such coherence be measured? In an effort to make an optical analogy [9], the difference between a coherent and a non-coherent state can be expressed as a spectral width. Such spectral width is inversely proportional to the coherence length and greater coherence guarantees greater interference and greater state purity. In the nuclear model, the coherence level of states follows a similar rule: lower configuration mixing (or greater purity) implies greater coherence. To follow this optical analogy, it is possible to define the visibility of the energy distribution as:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (5)$$

where I_{max} and I_{min} are referring to the intensities (or counts numbers) from a peak to a consecutive valley in a distribution.

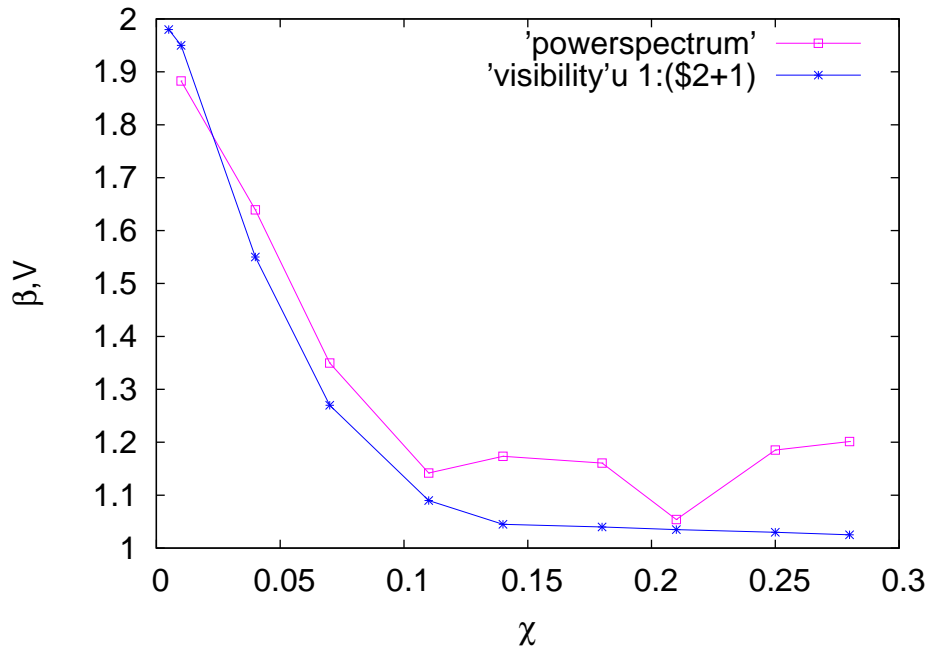


Figure 3. Visibility and power spectra parameter for energy time series dependent of the quadrupole-quadrupole parameter χ . In order to compare, visibility was increased by one.

Figure 3 shows both the visibility (asterisks) and power spectrum parameter (squares) as functions of the quadrupolar interaction. As visibility is related to state purity, it is possible to say that state mixing increases exponentially and that half of the total mixing is attained for a quadrupolar interaction of $\chi = 0.045$. Other interesting aspect is that for a standard quadrupolar interaction ($\chi = 0.21$) the visibility is around 5%. We can then ask ourselves if a visibility of 5% only represents noise or if there still is some degree of correlation. In order to investigate this point, we compare with the power spectra fluctuation analysis. By subtracting the mean behavior (in all curves in Fig. 1) we can obtain the energy time series of the fluctuations. In [10, 11] the method of power spectrum analysis (PS) [14, 15, 16, 17] is used to study the statistical of fluctuations of energy time series. Following the $1/f^\beta$ noise convention, three of the most conventional noises present in nature are for $\beta = 2, 1, 0$ corresponding to thermal (Brownian), chaotic and Poisson noise respectively [12]. PS curve shows the behavior of the noise analyzed with the power spectrum method as a function of the quadrupolar interaction parameter: β vs. χ . For low values of χ , the noise appears to be $1/f^2$, and it evolves to a $1/f$ noise as interaction increases. There is a clear similarity between noise behavior and visibility: both show an exponential regime in the same $0, 0.1 \chi$ interval.

It is also interesting to remark that the statistics of the time series is shifted to a $1/f$ noise, corresponding to approximately a visibility of 5%. It is thus tempting to conjecture that a zero visibility would correspond to a $1/f^0$ noise, or a random noise. Although in reference [13] may be argued otherwise, we never find any random noise. With the convention that 100% visibility corresponds to totally pure states with a maximal coherence length (all the states of the ensemble correspond to one single state ψ^i of the base), zero visibility would correspond to the states of an ensemble with a null coherence length represented by a random mixing of all the states of the base. Hence, time series with interactions such that $\chi > 0.1$, still possess a certain degree of coherence. In this case, the $1/f$ noise, characteristic of

nuclear spectrum [14, 15, 16, 17, 10], corresponds to the same fluctuations found in the analysis of the eigenvalues of random matrices [18, 19, 20] of a two-body random ensemble (TBRE). Studies of these random matrices showed that nuclear energy levels of low-energy excited states of complex nuclei follow a complete statistics, and not a random one. This complexity is called quantum chaos. We can propose that such fluctuations still possess information, given the correspondence. This information is stored in the remaining coherence that originates the 5% visibility.

5. Decoherence and conclusions

To analyze this remaining coherence, the dependence of eigenvectors with the quadrupolar interaction parameter has been studied. Figures 4 and 5 show the distribution of the squares of the 1627 quantum components ($|C_K^\alpha|^2$) of the 1627 states $|\psi_K\rangle$ with $J^p = 3^+$ in ^{48}Ca for $\chi = 0.01$ and $\chi = 0.21$ respectively. It can be observed that in the first case (fig.4) the components distribution is fractioned, the states are still organized in groups. As previously discussed, the states are locally mixed for this level of interaction. In other words, we can say that components of a given states group have not mixed with components of another group. However, as interaction increases the groups vanish showing a more homogeneous mixing (fig.5). It is worth mentioning that the envelope of such a distribution presents similarities with the information distribution obtained from the entropy $S(\alpha) = -\sum_K |C_K^\alpha|^2 \log |C_K^\alpha|^2$ where ψ^α is the state described in the $|K\rangle$ base states [21], [22]. The shape of this global distribution tells us that the states furthest from the energy mean are very close to what could be considered as pure states: only very few components are present in the superposition. The particular case of the 3^+ state of less energy possess around ten components, corresponding to a degree of purity (seen geometrically) of approximately 94% ($[1 - (10/1627)]100$). As the state-mixing increases, the number of components involved in the description of the state increases. The principal components form a group with a maximum number in the central states.

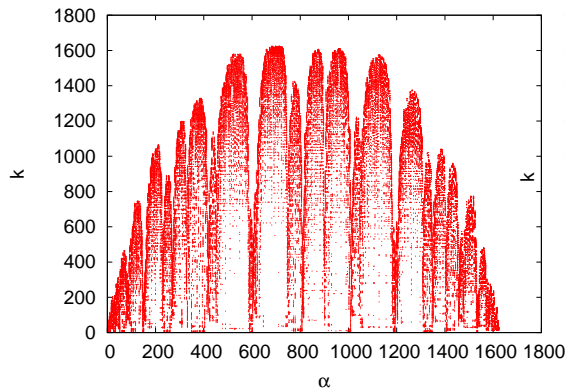


Figure 4. Distribution of probabilities $|C_k^\alpha|^2$ for all 1627 α states. For $\chi = 0.005$ it is appreciable the interference among basis states.

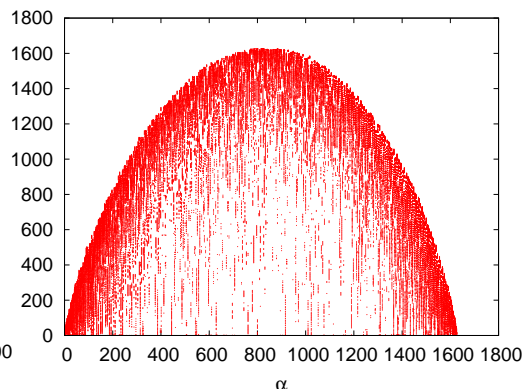


Figure 5. Similar distribution like Fig. 4, for $\chi = 0.18$. The interference pattern is almost erased.

In conclusion, we have studied the statistics of energy levels considering the visibility of its distribution as a function of the parameter of quadrupolar interaction. Visibility is an indication of the degree of coherence of nuclear states associated to studied energies. By comparing the visibility to the respective power spectrum, we found a correspondence between the statistics of energy levels and the degree of purity of the nuclear states. The $1/f$ fluctuations characterizing

quantum chaos may be interpreted as a residuum of coherence between nuclear states. The lowest states maintain a high degree of purity, whereas decoherence has almost erased the origin of all intermediate energy states. To define the behavior known as quantum chaos, it is therefore not necessary to manipulate classical or semi-classical images from some dynamic system. Quantum decoherence may be used instead.

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